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Mathematical Analysis of Machine repair problem with common cause failure, hot spares and multiple repairmen

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Abstract: *We study the machine repairable system comprising M operating machines, H spares and more than one repairman where "the partial server vacation" is applied on some of the repairmen. In this system, the first repairman never takes vacation and always available for servicing of failed machines while other repairmen goes to random length vacation whenever the number of failed machines are less than N, N +1 respectively. Machines may breakdown individually or due to common cause according to Poisson process. Vacation time and service time of repairmen is follows the exponential distribution. Recursive approach is used to obtain the steady state probabilities. A cost model is developed to determine the optimum value of failed machine maintaining the system availability. Sensitivity analysis is investigated for optimal conditions and also analyzes the reliability characteristics of the system.*

Keywords: Repairable system; Spares; Partial server vacation; Common cause failure; Sensitivity analysis.

1. Introduction

In this paper, we deal with a machine repairable system comprising operating and spare machines where failed machines are repair by repairmen in the repair facilities under the partial server vacation policy. Generally, in multiple server systems, only some of servers perform secondary jobs, during their idle period and the other servers are always available for servicing the failed machines. Such type of vacation is known as "the partial server vacation". A spare is known as "hot spare" if its failure rate is the same as of an operating machine. Many researchers have shown their interest in study of machine repairable system using the spares since long. Instead of going in detail of such previous works, it is worth-while to give a brief overview of recent past contribution of researchers.

Sharma (2011) consider a repairable system with spares consisting two failure modes and introduced the concept of spare. In case of any sudden cause, available spares immediately substituted to failed machine. A repairable machining system with provision of spares has also been studied by Wang and Sivzlian (1992), Sharma (2012), Jharotia (2015) etc. Replacement of a machine is always not possible and nor required, availability of repairmen prevent the replacement. Wang and Sivzlian (1989) have studied the reliability characteristic of repairable system using S warm spares, M operating machines and R repairmen. The expression for reliability of system and mean time taken until the first failure obtained by them. Chien (2010) introduced a model to obtain the optimal number of repair before using spares for prevention of replacement, under some conditions; he has shown that optimal number of repair is required for minimizing the system cost. Ke et al. (2013) examined a machine repair problem with unreliable multi-repairmen and introduced the concept of coverage probability for failed machine and proposed Quasi-Newton method to obtain optimum number of repairmen as well as spares. Wang and Ke (2003) extend it to incorporating balking and reneging of failed machine.

To reduce the burden in terms of cost of a system, repairmen can do secondary job. Some of excellent surveys including some of applications can be found in Doshi (1986), Tian & Zhang (2006) and Gupta (1997). Ke (2006) extended the work of Gupta (1997) for unreliable server. Machine repairable problem with two types of spares and various server vacation policies was studied by Ke and Wang (2007). Direct search algorithm is used to get optimum number of standbys as well as servers and to established steady state probabilities by using matrix geometric approach. Ke and Wu (2012) investigated the concept of synchronous multiple vacation in multi server machine repair model with standbys. Yue et al. (2012), Sharma (2012) and Jharotia & Sharma (2015) also studied a machine repairable system using the concept of partial server vacation. They studied with the view point of queuing and reliability for machine repairable system with two repairmen and warm spares. Longshree et al. (2015) studied a machine repairable system with two repairmen, hot spares and partial server vacation policy. They discussed a cost model to obtain the optimal value of failed machine. The common cause failures have a major impact on the availability and reliability of repairable system. Several situations such as humidity, shock voltage fluctuation, temperature etc. that prevail in many machining system and other applications can cause simultaneous failure of some or all machines of the system. The Research works in this area have been done by Dai and Wang (2007), Hughes (1987), Jain and Mishra (2006), Kvam and Miller (2002) etc worked on common cause failures. The present work is different from all above works in the sense that we are introducing three repairmen with hot spares and common cause failure.

The objective of this paper is as follows. In section 2, we first provide the details of the model. We formulate the problem and computed steady state probability for number of failed machine using recursive approach in section 3. The expressions of some performance measures for queuing and reliability are obtained in section 4; those are similar to Dequan (2012), Sharma (2012) and Longshree (2015). In section 5 & 6, constructed a cost model to find the optimum number of failed machine and perform sensitivity analysis for investigating the effect of system parameters on the optimal cost. In section 7, conclusion of the work has given.

2. System Model

We have considered a machine repairable system with M operating machines, H hot spares and three repairmen in the repair facility. The first repairman never takes vacation and always available for servicing of failed machines while second and third repairmen goes on vacations under certain conditions (partial server vacation). We call the first repairman by “R1”, the second repairman by “R2” and the third repairman by “R3”. The assumptions for development of model and constructing the governing equations are similar to Dequan (2012), Sharma (2012) and Longshree (2015). Assumptions for the system model as follows:

- Initially M-operating machines are required to start the system. However, the system can also work in short mode when all spare machines are exhausted and number of operating machines are less then M but more than K operating machines. In other words, the system will not work if number of failed machines are L = M+H-K+1 or more than L. The failure rates of spare as well as operating machine are exponentially distributed with rates of λ, means that both types of machines have same failure rate due to using hot spares.
- The operating machines may also fail due to common cause according to possion process with the rate λ_c. The switch over time from repair to operating or standby group is negligible.
- Whenever an operating unit fails, it’s immediately replaced by an available spare and failed machine sent for repair at repair facility. After repair of a failed machine, it is as good as new one and can join operating state if operating machines are less than M, otherwise goes into standbys group. The repair time of all repairmen (R1, R2 and R3) are exponentially distributed with service rate μ_1, μ_2 and μ_3, respectively.
- The vacation period of repairmen is exponentially distributed with parameter ν.
- The mean failure rate is given as follows

$$\lambda_n = \begin{cases} (\mathrm{M}+\mathrm{H}-\mathrm{n})\lambda + \lambda_c, & n = 0,1 \ldots L-1 \\ 0, & \mathrm{n} = \mathrm{L} \end{cases}$$

3. Steady-state Analysis

The steady-state probabilistic equations of the system model are established with the help of transition diagram. We have adopted a recursive approach to obtain the expressions of the steady state probabilities and we also give the explicit expression of some performance measures. Let $L(t)$ be the number of failed machines in the repair facility at time t and state of the server denoted by $J(t)$, defined as follows -

$$J(t) = \begin{cases} 0, & \text{R2 and R3 both on vacation} \\ 1, & \text{R3 on vacation} \\ 2, & \text{No Repairmen on vacation} \end{cases}$$

Let (L (t), J (t)) is a Markov process with state space

E = {(n, 0): n = 0, 1 ... L} U {(n, 1): n = N, N + 1...L} U {(n, 2): n = N + 1, N + 2 ... L}

The steady-state probability distributions of the system define as follow:

$$P_{n,0} = \lim_{t\to\infty} P\{L(t) = n, \quad J(t) = 0\}, \qquad 0 \le n \le L$$

$$P_{n,1} = \lim_{t\to\infty} P\{L(t) = n, \quad J(t) = 1\}, \qquad N \le n \le L$$

$P_{n,2} = \lim_{t\to\infty} P\{L(t) = n, \quad J(t) = 2\}, \qquad N+1 \le n \le L$

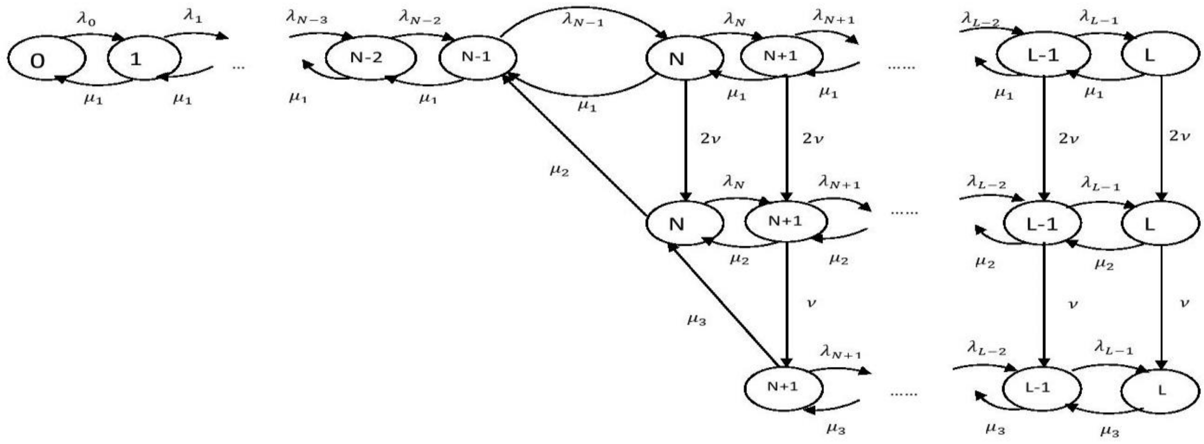


Figure 1: Transition state diagram

With the help of transition diagram, governing equations of the model obtained as follow:

$$-\lambda_0 P_{0,0} + \mu_1 P_{1,0} = 0 \tag{1}$$

$$\lambda_{n-1}P_{n-1,0} + \mu_1 P_{n+1,0} - (\lambda_n + \mu_1)P_{n,0} = 0, \qquad 1 \le n \le N-2, \tag{2}$$

$$\lambda_{N-2}P_{N-2,0} + \mu_1 P_{N,0} + \mu_2 P_{N,1} - (\lambda_{N-1} + \mu_1)P_{N-1,0} = 0 \tag{3}$$

$$\lambda_{n-1}P_{n-1,0} + \mu_1 P_{n+1,0} - (\lambda_n + \mu_1 + 2\nu)P_{n,0} = 0, \qquad N \le n \le L-1, \tag{4}$$

$$\lambda_{L-1}P_{L-1,0} - (\mu_1 + 2\nu)P_{L,0} = 0 \tag{5}$$

$$2\nu P_{N,0} + \mu_2 P_{N+1,1} + \mu_3 P_{N+1,2} - (\lambda_N + \mu_2)P_{N,1} = 0 \tag{6}$$

$$2\nu P_{n,0} + \mu_2 P_{n+1,1} + \lambda_{n-1}P_{n-1,2} - (\lambda_n + \mu_2 + \nu)P_{n,1} = 0, \quad N+1 \le n \le L-1 \tag{7}$$

$$2\nu P_{L,0} + \lambda_{L-1}P_{L-1,1} - (\mu_2 + \nu)P_{L,1} = 0 \tag{8}$$

$$\nu P_{N+1,1} + \mu_3 P_{N+2,2} - (\lambda_{N+1} + \mu_3)P_{N+1,2} = 0 \tag{9}$$

$$\nu P_{n,1} + \lambda_{n-1}P_{n-1,2} + \mu_3 P_{n+1,2} - (\lambda_n + \mu_3)P_{n,2} = 0, \qquad N+2 \le n \le L-1, \tag{10}$$

$$\nu P_{L,1} + \lambda_{L-1}P_{L-1,2} - \mu_3 P_{L,2} = 0 \tag{11}$$

With the normalizing condition

$$\sum_{n=0}^{L} P_{n,0} + \sum_{n=N}^{L} P_{n,1} + \sum_{n=N+1}^{L} P_{n,2} = 1 \tag{12}$$

In order to solve the steady- state probability equations (1) – (11), we define

$$\varphi_n = \begin{cases} (\frac{1}{\mu_1})^n \prod_{j=0}^{n-1} \lambda_j, & 1 \le n \le N-1 \\ (\frac{1}{\mu_1})^{N-1} \prod_{j=N}^{n} \beta_j \prod_{j=0}^{N-2} \lambda_j, & N \le n \le L \end{cases} \tag{13}$$

Where $\beta_j, j = N, N+1 \ldots L$ is defined iteratively as follows:

$$\beta_n = \begin{cases} \frac{\lambda_{n-1}}{\mu_1 + 2\nu\left[1 + \sum_{i=n+1}^{L} \prod_{j=n+1}^{i} \beta_j\right]}, & N \le n \le L-1 \\ \frac{\lambda_{L-1}}{(\mu_1 + 2\nu)}, & n = L \end{cases} \tag{14}$$

The solutions of steady-state probabilities equations established through given theorem.

Theorem: The steady-state probabilities of the system are given as follows

$$P_{n,0} = \varphi_n P_{0,0}, \qquad 1 \le n \le L \tag{15}$$

$P_{n,1} = \frac{2\nu}{\mu_2}\psi_n P_{0,0}\,, \qquad N \le n \le L$ (16)

$P_{n,2} = \frac{2\nu^2}{\mu_2\,\mu_3}\xi_n P_{0,0}\,, \quad N+1 \le n \le L$ (17)

and

$P_{0,0} = \left(1 + \sum_{n=1}^{L} \varphi_n + \frac{2\nu}{\mu_2}\sum_{n=N}^{L}\psi_n + \frac{2\nu^2}{\mu_2\mu_3}\sum_{n=N+1}^{L}\xi_n\right)^{-1}$ (18)

Where φ_n defined in eq. (13), ψ_n and ξ_n defined as

$$\psi_n = \begin{cases} \sum_N^L \varphi_i\,, & n = N \\ \frac{\lambda_{n-1}}{\mu_2}\psi_{n-1} + \sum_{i=n}^{L}\varphi_i - \frac{1}{2}\sum_{i=n}^{L}\psi_i\,, & N+1 \le n \le L \end{cases} \quad (19)$$

$$\xi_n = \begin{cases} \sum_{i=N+1}^{L}\psi_i\,, & n = N+1 \\ \frac{\lambda_{n-1}}{\mu_3}\xi_{n-1} + \sum_{i=n}^{L}\psi_i\,, & N+2 \le n \le L \end{cases} \quad (20)$$

Proof: From Eq. (1) and (2)

$$P_{n,0} = \frac{\lambda_{n-1}}{\mu_1} P_{n-1,0} \qquad 1 \le n \le N-1 \quad (21)$$

by this recursive relation, we get a relation

$$P_{n,0} = \varphi_n P_{0,0}, \qquad 1 \le n \le N-1 \quad (22)$$

where φ_n for $1 \le n \le N-1$ is defined by Eq. (13).

from Eq. (5)

$$P_{L,0} = \beta_L P_{L-1,0} \quad (23)$$

where β_L is defined by Eq. (14)

from Eqs. (4) and (5)

$$\lambda_{n-1}P_{n-1,0} - (\mu_1 + 2\nu)P_{n,0} = 2\nu\sum_{i=n+1}^{L} P_{i,0} \quad N \le n \le L-1 \quad (24)$$

By using Eq. (23) and (24) recursively, we get

$$P_{n,0} = \beta_n P_{n-1,0}, \qquad N \le n \le L-1 \quad (25)$$

where β_n is defined by Eq. (14)

from Eq. (22), (23) and (25), we get Eq. (15)

from Eq.(10)

$$\mu_3 P_{n,2} - \lambda_{n-1}P_{n-1,2} = \mu_3 P_{n+1,2} - \lambda_n P_{n,2} + \nu P_{n,1}, \qquad N+2 \le n \le L-1 \quad (26)$$

By repeating use of (26) and (11), we get

$$P_{n,2} = \frac{\lambda_{n-1}}{\mu_3}P_{n-1,2} + \frac{\nu}{\mu_3}\sum_{i=n}^{L} P_{i,1}, \qquad N+1 \le n \le L-1 \quad (27)$$

After taking n = N+2 in (27)

$$P_{N+2,2} = \frac{\lambda_{N+1}}{\mu_3}P_{N+1,2} + \frac{\nu}{\mu_3}\sum_{i=N+2}^{L} P_{i,1} \quad (28)$$

From (9) and (28)

$$P_{N+1,2} = \frac{\nu}{\mu_3}\sum_{i=N+1}^{L} P_{i,1} \quad (29)$$

From (7)

$$\mu_2 P_{n,1} - \lambda_{n-1}P_{n-1,1} = \mu_2 P_{n+1,1} - \lambda_n P_{n,1} + 2\nu P_{n,0} - \nu P_{n,1}, \quad N+1 \le n \le L-1 \quad (30)$$

By repeating use of (30) and (8), we get

$$P_{n,1} = \frac{\lambda_{n-1}}{\mu_2}P_{n-1,1} + \frac{2\nu}{\mu_2}\sum_{i=n}^{L} P_{i,0} - \frac{\nu}{\mu_2}\sum_{i=n}^{L} P_{i,1}, \qquad N+1 \le n \le L-1 \quad (31)$$

after taking n = N+1 in Eq.(31), we get

$$P_{N+1,1} = \frac{\lambda_N}{\mu_2}P_{N,1} + \frac{2\nu}{\mu_2}\sum_{i=N+1}^{L} P_{i,0} - \frac{\nu}{\mu_2}\sum_{i=N+1}^{L} P_{i,1} \quad (32)$$

From Eq. (6), (29) & (32)

$$P_{N,1} = \frac{2\nu}{\mu_2}\psi_N P_{0,0} \tag{33}$$

from Eq. (15), (31) and (33), we get

$$P_{n,1} = \frac{2\nu}{\mu_2}\psi_n P_{0,0}\ , N+1 \le n \le L \tag{34}$$

where ψ_n is define by Eq. (19)

From (29) & (33)

$$P_{N+1,2} = \frac{2\,\nu^2}{\mu_2\mu_3}\xi_{N+1} P_{0,0} \tag{35}$$

Substitute Eq. (27), (34) and (35)

$$P_{n,2} = \frac{2\,\nu^2}{\mu_2\mu_3}\xi_n P_{0,0}, \quad N+2 \le n \le L \tag{36}$$

where ξ_n defined in Eq. (20)

Substitute Eq. (15), (16) and (17) into (12), we get

$$P_{0,0} = \left(1 + \sum_{i=1}^{L}\varphi_i + \frac{2\nu}{\mu_2}\sum_{i=N}^{L}\psi_i + \frac{2\,\nu^2}{\mu_2\mu_3}\sum_{i=N+1}^{L}\xi_i\right)^{-1}$$

4. Performance Measures

Using steady state probabilities given in previous section, we can easily compute queuing and reliability measure of machine repair problem, which is given as follows.

i) The expected number of failed machines in the system is given by

$$E_N = \left(\sum_{n=1}^{L} n\varphi_n + \frac{2\nu}{\mu_2}\sum_{n=N}^{L} n\psi_n + \frac{2\,\nu^2}{\mu_2\mu_3}\sum_{n=N+1}^{L} n\xi_n\right)P_{0,0}$$

ii) The expected number of spare machines in the system is given by

$$E_S = \begin{cases} \left(\sum_{n=0}^{H-1}(H-n)\varphi_n + \frac{2\nu}{\mu_2}\sum_{n=N}^{H-1}(H-n)\psi_n + \frac{2\nu^2}{\mu_2\mu_3}\sum_{n=N+1}^{H-1}(H-n)\xi_n\right)P_{0,0}, & N \le H \\ \left(\sum_{n=0}^{H-1}(H-n)\varphi_n\right)P_{0,0}, & N > H \end{cases}$$

iii) The probability that R1 is busy is given by

$$P_{b1} = 1 - P_{0.0}$$

iv) The Probability that R2 is busy is given by

$$P_{b2} = \frac{2\nu}{\mu_2}\sum_{n=N}^{L}\psi_n P_{0,0}$$

v) The Probability that R3 is busy is given by

$$P_{b3} = \frac{2\nu^2}{\mu_2\mu_3}\sum_{n=N+1}^{L}\xi_n P_{0.0}$$

vi) The steady-state availability of the system is given by

$$A_s = 1 - \left(\varphi_L + \frac{2\nu}{\mu_2}\psi_L + \frac{2\nu^2}{\mu_2\,\mu_3}\xi_L\right)P_{0.0}$$

vii) The fraction of total time that the machines are working (machine availability) is given by

$$M.A. = 1 - \frac{E_N}{M+H}$$

5. Cost Analysis

We construct a cost function to determine the optimum number of failed machine (N) to maintain the system availability for working state. The optimum value denote by N* that is obtained by minimizing the cost function. Parameters of our model are similar to Ke and Wang (2007). Let

C_N - Cost per unit time of one failed machine in repair facility

C_S - Cost per unit time of one machine that functions a spare

C_{b1}- Cost per unit that R1 is busy

C_{b2} - Cost per unit that R2 is busy

C_{b3} - Cost per unit that R3 is busy

C_I - Cost per unit time that R1 is idle

C_{R2} - Reward per unit time that R2 is on vacation and

C_{R3} - Reward per unit time that R3 is on vacation.

With the costs defined above and machine availability requirements, we consider N is a decision variable. Total expected cost function as follows

$$F(N) = C_N \mathrm{E_N} + C_S E_S + C_{b1}P_{b1} + C_{b2}P_{b2} + C_{b3}P_{b1} + C_I P_I - C_{R2}P_{R2} - C_{R3}P_{R3}$$

Where

$P_I = P_{0,0}$ - The probability is that R1 is idle,

($P_{R2} = 1 - P_{b2}$) - The probability is that R2 is on Vacation,

($P_{R3} = 1 - P_{b3}$) - The probability is that R3 is on Vacation,

E_N , E_S , P_{b1} , P_{b2} *and* P_{b3}, are given by above statement.

To ensure the availability of the system at working state, the cost minimization problem as follows:

$$\mathrm{Min}\, F(N) = C_N \mathrm{E_N} + C_S E_S + C_{b1}P_{b1} + C_{b2}P_{b2} + C_{b3}P_{b1} + C_I P_I - C_{R2}P_{R2} - C_{R3}P_{R3}$$

$$s.t\ \ A_s \geq A_0$$

Where, A_s is the steady state availability of the system given in performance measures and A_0 is the pre-specified level of system availability

6. Sensitivity investigation

In this section, we perform a sensitivity analysis to explore the effects of changes in system parameters on optimum cost of the system $F(N *)$. It is difficult to analysis the system by analytic method. To determine the optimum number of failed machines (N*), so we may use a heuristic approach by satisfying inequalities as follows:

$$F(N^* - 1) > F(N^*) < F(N^* + 1)$$

$$\text{And } A_s \geq A_0$$

6.1 To find optimum value of failed machine (N*)

For computing purpose, we consider M = 10, H = 6, K = 5 and $A_0 = 0.9$ and cost values as follows:

$$C_N = 80, C_S = 70\ , C_{b1} = 70, C_{b2} = 60,\ C_{b3} = 50\ C_I = 40, C_{R2} = 70, C_{R3} = 90$$

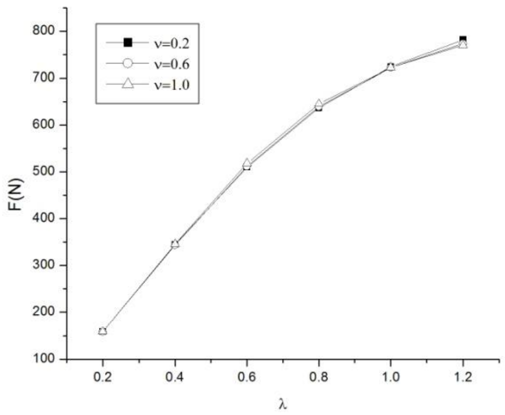
The numerical results of value of N, performance measure and the total expected cost of system is given in table-1. It shows the optimal number of failed machine (N*) = 4, optimum cost of system F (N*) = 345.84, availability of the system A_s = 0.9997 and machine availability (MA) = 0.8216 by fixing $\lambda = 0.4,\ \lambda_c = 0.02,$ $\mu_1 = 6,\ \mu_2 = 8, \mu_3 = 10,\ \nu = 1.$

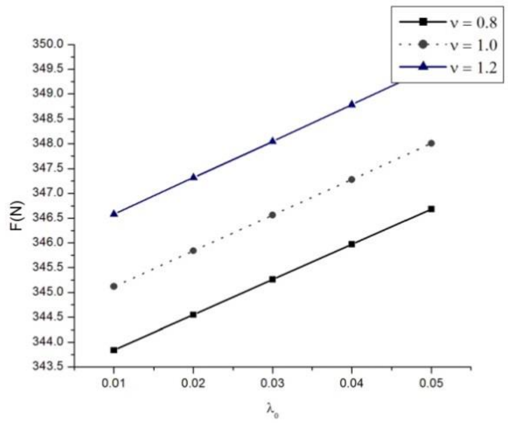
$Table\ 1.\ The\ expected\ cost, system\ performance\ measures\ and\ number\ of\ failed\ machine\ for$

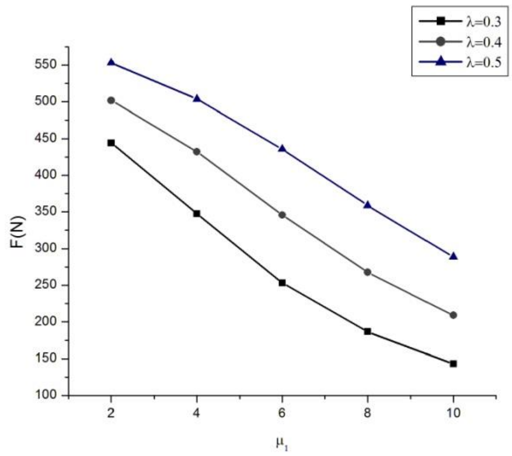
$\lambda = 0.4,\ \lambda_c = 0.02,\ \mu_1 = 6,\ \mu_2 = 8, \mu_3 = 10,\ \nu = 1$

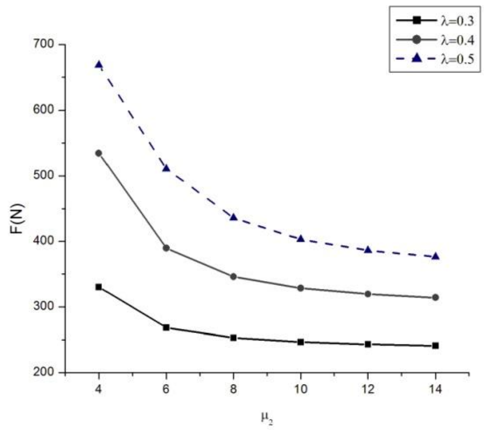
N	En	Es	Pb1	Pb2	Pb3	PI	PR2	PR3	MA	As	FN
1	2.4995	2.4346	0.8103	0.3812	0.1682	0.1896	0.6187	0.8317	0.8437	0.9998	348.50
2	2.6197	2.4151	0.8250	0.2945	0.1094	0.1749	0.7054	0.8905	0.8362	0.9998	346.96
3	2.7396	2.3746	0.8358	0.2171	0.0678	0.1641	0.7828	0.9321	0.8287	0.9997	346.05
4	**2.8529**	**2.3242**	**0.8435**	**0.1517**	**0.0397**	**0.1564**	**0.8482**	**0.9602**	**0.8216**	**0.9997**	**345.84**
5	2.9534	2.2738	0.8488	0.0996	0.0217	0.1511	0.9003	0.9782	0.8154	0.9996	346.21
6	3.0360	2.2304	0.8524	0.0608	0.0110	0.1475	0.9391	0.9889	0.8102	0.9996	346.98
7	3.0982	2.1973	0.8546	0.0342	0.0050	0.1453	0.9657	0.9949	0.8063	0.9995	347.86
8	3.1408	2.1783	0.8558	0.0174	0.0020	0.1441	0.9825	0.9979	0.8036	0.9994	348.81
9	3.1668	2.1683	0.8565	0.0079	7.23E-4	0.1434	0.9920	0.9992	0.8020	0.9993	349.57
10	3.1809	2.1635	0.8568	0.0031	1.95E-4	0.1431	0.9968	0.9998	0.8011	0.9992	350.04

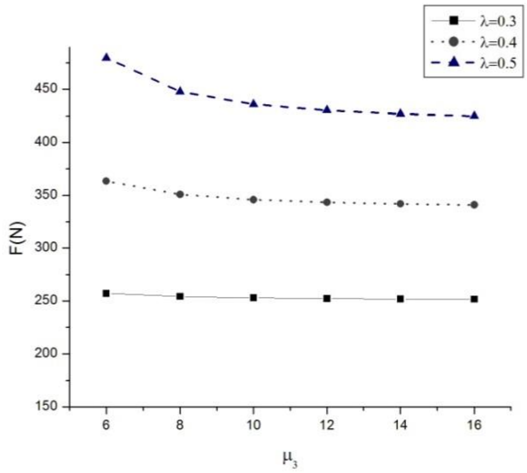
6.2 Effect of system parameters on optimum cost $F(N)$

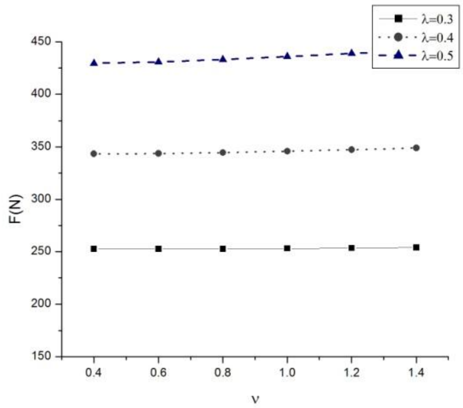












In fig. 2, given below, we fix $\lambda_c = 0.02,\ \mu_1 = 6,\ \mu_2 = 8,\ \mu_3 = 10$ the three curves are corresponding to ν = 0.2, 0.6, 1.0 respectively. This figure shows that optimum cost increase significantly as ν increases whereas it almost constant by increasing the vacation rate ν. This is because the expected number of failed machines increases as λ increases whereas it decreases as service rate increases. From figure It is very clear that three curve are almost overlapped when λ increases from 0.1 to 1.2. In fig. 3, we fix $\lambda = 0.4$, $\mu_1 = 6,\ \mu_2 = 8,\ \mu_3 = 10$ the three curves are corresponding to ν = 0.8, 1.0, 1.2 respectively. This figure shows that the optimum cost directly proportional to λ_c.

In fig. 4, we fix $\lambda_c = 0.02,\ \mu_2 = 8,\ \mu_3 = 10, \nu = 1$, the three curves are corresponding to λ = 0.3, 0.4, 0.5 respectively. This figure shows that the optimum cost decrease significantly as μ_1 increases whereas it increases by increasing the failure rate λ. This is because the expected number of failed machines decreases as μ_1 increases whereas it increases as λ increases. In fig. 5, we fix $\lambda_c = 0.02,\ \mu_1 = 6,\ \mu_3 = 10, \nu = 1$, the three curves are corresponding to λ = 0.3, 0.4, 0.5 respectively. This figure shows that the optimum cost decrease as μ_2 increases whereas it increases slightly by increasing the failure rate λ. In fig. 6, we fix $\lambda_c = 0.02,\ \mu_1 = 6,\ \mu_2 = 8, \nu = 1$, the three curves are corresponding to λ = 0.3, 0.4, 0.5 respectively. This figure shows that the optimum cost decrease significantly as μ_3 increases whereas it increases by increasing the failure rate λ. By fixing $\mu_1 = 6,\ \mu_2 = 8, \mu_3 = 10$ in fig. 7, we get the three curves are corresponding to λ = 0.3, 0.4, 0.5 respectively. This figure shows that the optimum cost almost constant as ν increase significantly by increasing the failure rate λ. This result shows that optimum cost is not affected by vacation rate.

7. Conclusions

In this paper, we deal with a machine repair system comprising operating machine and hot spare where failed machines are repaired by repairmen in the repair facilities under the partial server vacation policy. Expression of the steady state probability is obtained by recursive approach. Performance measure for queuing and reliability have been developed to analysis the system characteristics and also perform sensitivity analysis. We also obtained the optimum value of failed machines N with the help of a cost model, for which the cost is minimum and availability of the system ensure its minimum level ($A_0 = 0.9$). We analyzed the effect of change in system parameters on system cost and find optimum values of system parameters as ($\lambda = 0.4,\ \lambda_c = 0.02,\ \mu_1 = 6,\ \mu_2 = 8,\ \mu_3 = 10$ and $\nu = 1$) with optimum cost F (N*) = 345.84.

This type of situation is very much beneficial to system engineers and helpful to reduce burden of system and making proper utilization of resources. This type of situation is observed in production and manufacturing system and transportation systems. Analysis of machining system with server vacation helps to study the impact of secondary jobs on performance of the system. The concept of spares and repairmen can helps to decision makers to design a cost effective system. The common cause failure can be seen in computer and communication systems.

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